Resonant splitting of a vector soliton in a periodically inhomogeneous birefringent optical fiber

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We analyze the dynamics of a two-component (vector) soliton in a model of a birefringent nonlinear optical fiber with a periodic spatial modulation of the birefringence parameter (group velocity difference). Evolution equations for the parameters of the vector soliton are derived by means of a variational technique. Numerical simulations of these equations demonstrate that the critical modulation amplitude necessary for splitting, regarded as a function of the soliton's energy, has a deep minimum very close to the point at which direct resonance takes place between the periodic modulation and an internal eigenmode of the vector soliton in the form of small relative oscillations of the centers of the two components. A shallower minimum, which can be related to another internal eigenmode of the vector soliton, is also found. We further briefly consider the internal vibrations of the vector soliton driven by a constant force, which corresponds to the birefringence growing linearly with propagation distance. The effect predicted has practical relevance to ultrashort (femtosecond) optical solitons, and it can be employed in the design of fiber-optical logic elements.

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I. INTRODUCTION

The dynamics of two-component solitons in birefringent nonlinear optical fibers has attracted a great deal of attention for several reasons. The birefringence can affect the propagation of solitons in long-distance soliton-based optical communication lines [1]; it may find a potential use in all-optical logic elements (e.g., directional couplers [2,3]); and lastly, it gives rise to a number of challenging problems both for experimental studies [4] and for theoretical analysis (see, e.g., Refs. [5–11]). One of the central problems considered in these studies is the possibility of the splitting of an initial two-component soliton into single-component ones [3,10].

In the present work, we shall analyze the dynamics of a vector (two-component) soliton in an inhomogeneous fiber. Thus far, the effects of a randomly varying birefringence have been analyzed in models of inhomogeneous fibers [1,8]. We shall consider a model of a fiber with a regular periodic inhomogeneity, aiming to find an effective *resonant* mechanism for the splitting of the vector soliton into its constituent components. It should be possible to create an artificial regularly modulated birefringence, fabricating a fiber either with a periodically modulated elliptical cross section or with a periodically modulated twist. From the technological point of view, it could be a problem to fabricate a very long fiber, keeping in it the strictly periodic modulation of the birefringence. However, in what follows it will be demonstrated that the effect considered may be of practical importance only for ultrashort solitons, so that the fiber does not need to be extremely long. Another important result obtained below is that the resonance may have a conspicuous finite width, so that moderate random changes in the modulation should not be very critical for the implementation of this effect.

It is known that a vector soliton admits internal oscillations with certain eigenfrequencies. In Ref. [6], two eigenfrequencies have been found for a "symmetric" vector soliton, i.e., one with equal components in both polarizations. One eigenmode corresponds to *positional* internal oscillations of the vector soliton, i.e., oscillations of the centers of the two components relative to each other (this eigenmode was also considered in Ref. [11]). The other eigenmode corresponds to *shape* oscillations in which the width and amplitude of the soliton oscillate synchronously in both components. Actually, this *symmetric* mode of the shape oscillations is the same as that found previously for a single-mode soliton [12]. Then, in Ref. [9] it has been demonstrated that the vector soliton has in addition a third eigenmode corresponding to *antisymmetric* shape oscillations, in which the width-amplitude oscillations of the two components (polarizations) are π out of phase relative to each other. In Ref. [9], it has also been demonstrated that there exists a continuous family of *asymmetric* stationary vector-soliton states, i.e., those with unequal components in the two polarizations. In the general case, the asymmetric soliton also has three eigenfrequencies of internal oscillations, the corresponding eigenmodes mixing the positional and shape oscillations.

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The existence of the eigenfrequencies suggests the possibility of resonant effects in a periodically inhomogeneous fiber when its periodicity is commensurate with one of the soliton's eigenfrequencies. Actually, all the eigenfrequencies are functions of a single parameter, viz., the energy of the soliton. Thus resonance can be expected at a certain value of the energy. Recently, we considered a similar problem for a single-mode model with a periodic inhomogeneity of the dispersion coefficient [13] (models equivalent to that used in Ref. [13] have been derived for a fiber with a gradient-index structure [14] and for a fiber with a periodically distributed gain compensating dissipative losses [15]). The dispersion coefficient was taken in the form

$$D(z)=1+\epsilon\cos(kz)\ . \tag{1}$$

where z is the propagation distance along the fiber and k is the wave number of the periodic modulation. In this case, resonance was possible with the shape oscillations of the single-mode soliton. It has been demonstrated that, in near-resonant cases, even a small inhomogeneity amplitude ϵ (typically $\epsilon\sim 0.05$) rendered the shape oscillations seemingly chaotic, and there always existed a critical value ϵ_{cr} at which the soliton was completely destroyed by the periodic inhomogeneity (i.e., the width of the soliton was no longer oscillating, but permanently growing instead). In all of the cases considered in Ref. [13], ϵ_{cr} was close to $\frac{1}{4}$. Note that at these values of ϵ the total dispersion coefficient given by Eq. (1) remains everywhere positive, so that the destruction of the soliton had to be associated just with dynamical resonances, not with merely changing the sign of ϵ from focusing into defocusing. Lastly, it is relevant to mention that nonlinear optical fibers with a strongly inhomogeneous dispersion coefficient have recently been fabricated [18], which lends an additional physical relevance to the analysis given in Ref. [13].

In the present work, we shall analyze the dynamics of a vector soliton within the framework of the model based on coupled nonlinear Schrödinger equations:

$$iu_z+i\delta(z)u_t+\tfrac{1}{2}u_{tt}+(|u|^2+A|v|^2)u=0\ , \tag{2a}$$

$$iv_z-i\delta(z)v_t+\tfrac{1}{2}v_{tt}+(|v|^2+A|u|^2)v=0\ . \tag{2b}$$

Here u and v are the amplitudes of the two different polarizations, and the birefringence parameter δ (which measures the difference between the group velocities of the two polarizations) is assumed to be a periodic function of the propagation distance,

$$\delta(z)=\epsilon\sin(kz)\ , \tag{3}$$

cf. Eq. (1). In Eq. (3) we have dropped a possible constant part of δ because it can be eliminated by means of an evident phase transformation. The birefringence implies as well a phase velocity difference between the two polarizations. However, the corresponding terms, $+\zeta(z)u$ and $-\zeta(z)v$, which should be added to Eqs. (2a) and (2b), respectively (ζ is proportional to the phase velocity difference), can also be eliminated. Nevertheless, Eqs. (2) and (3) represent but the simplest model of a periodically inhomogeneous nonlinear fiber, since we have neglected the periodic z dependence of the other coefficients, as well as a possible *linear* coupling between the polarizations which occurs in a model of a periodically twisted fiber [16]. In spite of this, analysis of this simplest model is worthwhile, as our main goal is to predict a new effect which is expected to be qualitatively similar in more sophisticated models: *resonant splitting* of the vector soliton into two simple (polarized) solitons in the case when the spatial frequency of the periodic inhomogeneity is close to one of the internal eigenfrequencies of the vector soliton (and, first of all, when the resonant eigenfrequency is the one corresponding to the relative oscillations of the centers of the two components of the vector soliton).

For comparison, we shall also briefly consider the case in which the birefringence parameter δ in Eqs. (2) is a linear function of z, $\delta=\epsilon z$. In terms of the effective equations of motion for the essential parameters of the vector soliton (see below), the linearly growing birefringence gives rise to a constant driving force, while the periodically modulated birefringence induces a periodic force. We shall show that the effect produced by the constant force is quite trivial in comparison with the resonant splitting.

It is necessary to emphasize that, in physical units, the spatial periods of all the eigenmodes of small internal oscillations of the soliton are automatically of the same order of magnitude as the soliton's natural period [13]. In the resonant case, when the period of modulation coincides with that of the eigenmode, the fiber should be, at least, several periods long, in order to make the resonant effects visible. For the usual picosecond solitons, the natural period can be as large as several hundreds of kilometers, so that it is not relevant to consider resonance for solitons of this type. However, for ultrashort (femtosecond) solitons, which are now intensively studied, this period can be diminished to several dozens of meters, so that the resonant effects can be produced in a periodically inhomogeneous optical fiber of a reasonable length. Of course, another difficulty is that, in the notation adopted in Eqs. (2) and (3), the effective birefringence coefficient drops proportionally to the soliton's mean period. The natural birefringence of silica is known to be weak, so that one may conclude that the birefringence will play no role for ultrashort solitons. However, the artificial birefringence, produced, e.g., by an elliptical cross section of the fiber, can be rendered much stronger than the natural birefringence. Another important circumstance is that, as will be shown below, the amplitude of the modulation [the parameter ϵ in Eq. (3)] necessary for splitting is quite small near the resonance point, which may facilitate implementing the theoretical prediction in experiments.

The resonant splitting seems a rather interesting effect in itself; it may also find some practical applications. Indeed, the splitting of a two-component soliton into unbound components could be used as an elementary operation in all-optical logic. In that case, it is just the periodic spatial modulation of the fiber which can be used to induce this process. In what follows, it will be demonstrated that resonant splitting is much easier to induce than off-resonant splitting.

As for the coupling constant A in Eqs. (2), it is known that, generally speaking, one may get any value of A in the interval $\frac{2}{3} \le A \le 2$, depending on the value of the so-called ellipticity angle [3]. However, of most practical interest are the values corresponding to the edges of this interval: $A = \frac{2}{3}$ (linear polarizations) and $A = 2$ (circular polarizations). In this work, we will primarily concentrate on these two special values of A.

Our analysis will be based on a variational approach analogous to that developed for vector solitons in Ref. [6]: assuming a given wave form for the soliton, we will insert it into the full Lagrangian for Eqs. (2) to derive a system of ordinary differential equations (ODE's) governing the evolution of the arbitrary parameters of the chosen wave form. This is done below in Sec. II. In Sec. III, we consider the different resonances which can exist in this system. Then, in Sec. IV we present the results of direct numerical simulations of the ODE's. We adopt the following mode for the simulations: the spatial period of the modulation is kept constant, while the parameter corresponding to the initial energy of the soliton is varied (obviously, in an experiment it should be much easier to change the soliton's energy than the fiber's modulation period). The most important characteristic of the splitting is the critical (minimum) value ϵ_{cr} of the modulation amplitude ϵ at which the splitting takes place. In Sec. IV, we display the dependencies ϵ_{cr} vs the soliton's energy for the two above-mentioned physically interesting values of the coupling constant A, $A = \frac{2}{3}$ and $A = 2$. The dependencies clearly demonstrate deep minima at the values of the energy at which the period of small internal positional oscillations of the soliton is very close to the fixed modulation period, which is a direct manifestation of resonance. The accuracy with which the location of the minima is predicted by the resonance condition seems surprising, since the period of the internal positional oscillations was found for *small* oscillations, while splitting is a strongly nonlinear effect, formally corresponding to an infinitely large amplitude of the oscillations. It is noteworthy that, although the above-mentioned resonant minima are indeed deep and sharp enough, they nevertheless have a finite width (especially in the case $A = \frac{2}{3}$), which is probably a purely nonlinear effect. The finite width of the minima implies that the effect may be tolerant to moderate fluctuations of the modulation period and amplitude, which, as was already mentioned above, gives a much better chance to observe it experimentally. Weaker additional minima are also seen in the numerically found dependencies, which may be identified with higher resonances.

When, at a fixed value of the soliton's energy, the modulation amplitude ϵ increases from zero to the critical value, the simulations demonstrate that the regular internal oscillations of the vector soliton, driven by the periodic inhomogeneity, gradually become irregular (seemingly chaotic), and at the value $\epsilon = \epsilon_{cr}$, the vector soliton splits after several oscillations. Not surprisingly, at resonant values of the soliton's energy the oscillations become chaotic at values of ϵ much smaller than far from resonance. Actually, at some off-resonant energies splitting has not been observed at all.

It is relevant to emphasize that, while the results to be displayed below directly demonstrate that resonance accounts for the modulation-induced splitting of a vector soliton, the resonant character of the decay of the single-mode soliton considered in the previous work [13] was much less obvious, and in that case the resonance could furnish only a qualitative, but not quantitative, explanation of decay under the action of a periodic modulation. Another noteworthy distinction from the previous work is that, at a sufficiently large propagation distance, a single-mode soliton in a periodically modulated fiber must inevitably decay, if not through the resonant mechanism, then simply through the emission of radiation. In contrast to this, the splitting of a vector soliton in a periodically modulated birefringent fiber is not inevitable at all, since, at a very large propagation distance, the vector soliton may slowly decay into radiation, keeping the two components together. Thus the modulation-induced splitting is a more nontrivial effect.

II. VARIATIONAL EVOLUTION EQUATIONS FOR PARAMETERS OF THE VECTOR SOLITON

A variational approach to the analysis of the dynamics of solitons in single-mode nonlinear fibers was developed in Refs. [17] and [12], and it was then generalized to vector solitons in birefringent fibers in Refs. [6–8]. The approach is based on the Lagrangian for Eqs. (2), which can be averaged in the form $L = \int_{-\infty}^{+\infty} \mathcal{L}\, dz$, where the Lagrangian density is

$$\begin{aligned}\mathcal{L} = {} & \tfrac{1}{2} i (u_z u^* - u_z^* u + v_z v^* - v_z^* v) \\ & + i\delta(z)(u_t u^* - u_t^* u - v_t v^* + v_t^* v) \\ & - \tfrac{1}{2}(|u_t|^2 + |v_t|^2) + \tfrac{1}{2}(|u|^4 + |v|^4) + A|u|^2|v|^2 \, . \end{aligned} \tag{4}$$

The next step is to adopt a particular wave form (*ansatz*) for the vector soliton. Following Ref. [6], we will adopt the following ansatz, corresponding to symmetric modes in the two polarizations:

$$\begin{aligned}(u,v) = \eta \, \mathrm{sech}\left[\frac{t \mp y}{W}\right] \exp\Bigg[& \pm i\Omega(t \mp y) \\ & + i\frac{b}{2W}(t \mp y)^2 + i\sigma_{1,2}\Bigg] \, , \end{aligned} \tag{5}$$

where the upper and lower signs and the subscripts 1 and 2 refer to u and v, respectively; η^2 and W are the peak power and temporal width of the soliton (taken to be equal for both components), b is the so-called chirp parameter (which is also assumed equal for u and v), $2y$ is the separation between the centers of the two components, and 2Ω is the frequency difference between them. Then the ansatz (5) is inserted into the Lagrangian (4). After integration and varying the resultant averaged Lagrangian with respect to all the free parameters (which are assumed to be arbitrary functions of z), we arrive at a system of dynamical equations which is a straightforward generalization of that derived in Ref. [6]. It is convenient to represent this system as two coupled second-order equations for the variables y and W:

$$\frac{d^2y}{dz^2}=2AKW^{-2}F'(\aleph)+\epsilon k\cos(kz)\ , \tag{6a}$$

$$\frac{d^2W}{dz^2}=\frac{4}{\pi^2}\{W^{-3}-KW^{-2}-3AKW^{-2}[F(\aleph)+\aleph F'(\aleph)]\}\ , \tag{6b}$$

where the particular form (3) for $\delta(z)$ has been inserted, K is an integral of motion which takes an arbitrary positive value ($K\equiv\eta^2W$, i.e., K actually measures the energy of the soliton pulse), $\aleph\equiv 2y/W$, and

$$F(\aleph)\equiv\frac{\aleph\cosh\aleph-\sinh\aleph}{\sinh^3\aleph}\ . \tag{7}$$

The other variables can be expressed in terms of y and W as follows:

$$\Omega=\frac{dy}{dz},\quad b=\frac{dW}{dz}\ .$$

The variables $\sigma_{1,2}$ drop out of the analysis as well, as they did in Ref. [6]. In what follows, we will set $k\equiv 1$ in Eqs. (6), which can always be done by means of the obvious rescaling: $z\to kz$, $y\to k^{1/2}y$, $W\to k^{1/2}W$, $K\to k^{-1/2}K$, $\epsilon\to k^{-5/2}\epsilon$.

III. POSSIBLE RESONANCES IN THE DRIVEN SYSTEM

The dynamical equations (6) with $\epsilon=0$ have a fixed point with the coordinates [6]

$$y=0,\quad W=W_0\equiv(1+A)^{-1}K^{-1}\ , \tag{8}$$

which corresponds to the exact vector-soliton solution of the underlying equations (2). Equations (6) and (7) can be expanded in a vicinity of the fixed point (8), to yield the equations governing the corresponding small oscillations of the variables y and $w\equiv(W-W_0)/W_0$:

$$\frac{d^2y}{dz^2}=-\tfrac{16}{15}AKW_0^{-3}y+\tfrac{16}{5}AKW_0^{-3}yw+\tfrac{128}{63}AKW_0^{-5}y^3+\epsilon\cos x\ , \tag{9a}$$

$$\frac{d^2w}{dz^2}=-\frac{4}{\pi^2}(1+A)^4K^4w+\frac{32}{5\pi^2}AKW_0^{-4}y^2\ . \tag{9b}$$

In Eqs. (9) nonlinear terms of the lowest order have been kept.

Neglecting the nonlinear corrections and setting $\epsilon=0$, one obtains from Eqs. (9) two (spatial) eigenfrequencies for small oscillations [6]:

$$q_y^2=\tfrac{16}{15}A(1+A)^3K^4\ , \tag{10a}$$

$$q_W^2=\frac{4}{\pi^2}(1+A)^4K^4\ , \tag{10b}$$

the subscripts indicating that these eigenfrequencies are related to small oscillations of the variables y and W, respectively.

When $\epsilon\neq 0$, one can expect a resonance between the linearized y oscillations and the periodic drive [see Eq. (6a)] when their periods, equal to (in the notation adopted) $2\pi/q_y$ and 2π, respectively, coincide, i.e., at

$$K^{-4}=\tfrac{16}{15}A(1+A)^3\ . \tag{11}$$

Taking into account the cubic term in Eq. (9a), one can also expect a weaker resonance in the case in which $q_y=\frac{1}{3}$ [see Eq. (10)]. A more important nonlinear resonance is expected in the case in which $q_W=2$. Indeed, in this case the variable y is driven at the frequency 1 according to Eq. (9a), and the quadratic term in Eq. (9b) becomes, as can be seen, a resonant drive. According to Eq. (10b), this resonance happens at

$$K^{-2}=\frac{1}{\pi}(1+A)^2\ . \tag{12}$$

Another noteworthy case is that in which the double eigenfrequency $2q_y$ coincides with q_W, so that the two degrees of freedom are in resonance with each other, although, generally speaking, they do not resonate with the external drive. It follows from Eqs. (10) that this happens at a rather small value of A,

$$A=\frac{15}{16\pi^2-15}\approx 0.11\ , \tag{13}$$

with K arbitrary.

Lastly, one can expect a special situation (double resonance) in the case in which the condition (12) holds and simultaneously $q_y^2=1$. Obviously, this happens at the value of the coupling constant A given by Eq. (13), with K given by Eq. (12):

$$K^{-2}\approx 0.389\ . \tag{14}$$

It was stated above that physical values of A occur in the interval $\frac{2}{3}\le A\le 2$, so that the value (13) is not physically relevant. However, in the numerical simulations presented in the next section we will consider this case too, as it seems to be of a certain methodological interest.

IV. NUMERICAL SIMULATIONS

To analyze different resonant cases in detail, we solved numerically the full system of ODE's (6) with the function $F(\aleph)$ defined by Eq. (7). For initial values $y(0)$ and $W(0)$, we always took those corresponding to the point (8), and we always took zero initial values for dy/dx and dW/dx. The objective of the simulations was, first of all, to monitor how the driven oscillations of the separation y change with increase of the drive's amplitude ϵ at a fixed value of the soliton's energy K, and, what is most important, to find a critical value ϵ_{cr} at which the symmetric vector soliton (4) splits into simple solitons. Evidently, in terms of our approach, the splitting will manifest itself as a switch from oscillations of y to its permanent growth. Accurate detection of ϵ_{cr} required a long computation time, as, at ϵ slightly larger than ϵ_{cr}, the separation of the two components of the vector soliton commenced after a large number of oscillations with a slowly increasing amplitude, and, strictly speaking, the duration of this transient period diverges when ϵ is exactly equal to its critical value. Note that a similar criterion was used in Ref. [13] to identify the decay of the soliton into radiation in the single-mode system with a periodically modulated dispersion coefficient (in Ref. [13], the oscillations of the soliton's width were monitored).

Then, varying the value of K, we aimed to plot a dependence $\epsilon_{cr}(K)$. This was done for the two aforementioned special values of the coupling parameter A that are of basic physical interest: $A=\frac{2}{3}$ and $A=2$. The numerically found dependencies ϵ_{cr} vs K are depicted in Fig. 1. For both physical values $A=\frac{2}{3}$ and $A=2$, one can clearly see the resonant character of the dependencies. First, we shall discuss in detail the results for $A=2$, as this case seems simpler. For this case, Eq. (11) predicts the simplest *linear* resonance at $K\approx 0.363$. As is seen from Fig. 1(a), a minimum value of ϵ_{cr} for $A=2$ is attained at a value of K which is slightly larger. This minimum critical value is

$$\epsilon_{cr}^{(1)}(2)\approx 0.093 \ , \tag{15}$$

where the superscript 1 indicates the simplest (first) resonance and the argument 2 of ϵ_{cr} pertains to the value of A.

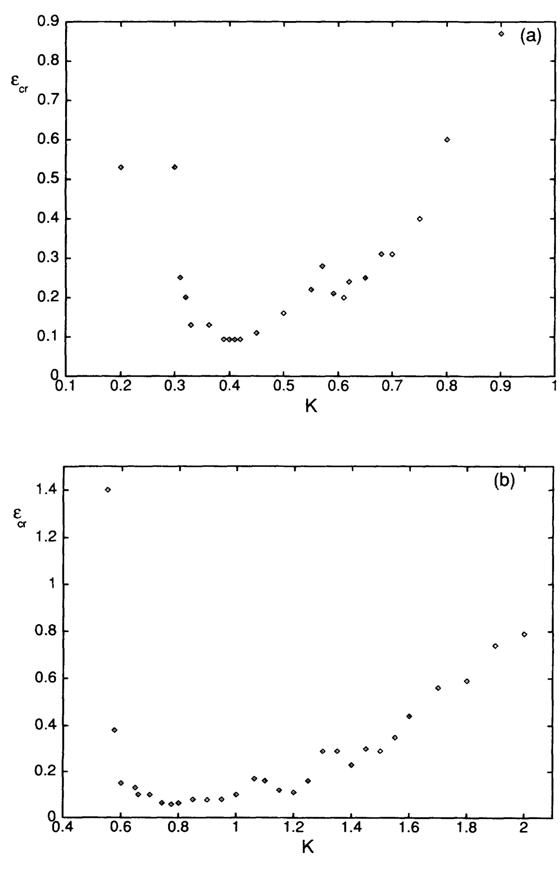

FIG. 1. The critical modulation amplitude, at which the vector soliton splits into components vs the soliton's energy K: (a) $A=2$; (b) $A=\frac{2}{3}$, A being the coupling constant in Eqs. (2).

The second resonance is predicted by Eq. (12) at $K\approx 0.591$. In the plot displayed in Fig. 1(a), we clearly see another minimum which is fairly close to this value. The value of ϵ_{cr} found at this point is

$$\epsilon_{cr}^{(2)}(2)\approx 0.20 \tag{16}$$

[cf. Eq. (15)], the superscript 2 indicating that this is the critical value at the point of the second resonance. In further simulations, higher resonances were not seen.

The accuracy with which the locations of the two minima are predicted by Eqs. (11) and (12) for $A=2$ is remarkable, as these expressions have been obtained from the *linearized* equations for small oscillations, while the splitting of the vector soliton is, patently, a strongly nonlinear effect (the amplitude of the oscillations diverges at the splitting point).

For $A=\frac{2}{3}$, Eq. (11) predicts the simplest resonance at $K\approx 0.742$. As is seen from Fig. 1(b) [note that the vertical scale in Fig. 1(b) is different from that in Fig. 1(a)], in this case we actually get a rather wide minimum in the dependence $\epsilon_{cr}(K)$, which begins practically exactly at the value predicted, and extends, approximately, up to $K=0.95$. The corresponding minimum value of ϵ_{cr} is [cf. Eq. (15)]

$$\epsilon_{cr}^{(1)}(\tfrac{2}{3})\approx 0.058 \ . \tag{17}$$

Then we get another minimum at K close to 1.2, with the value [cf. Eq. (16)]

$$\epsilon_{cr}^{(2)}(\tfrac{2}{3})\approx 0.11 \ . \tag{18}$$

Note that, in the present case, Eq. (12) predicts a second resonance at $K\approx 1.063$. It seems plausible that, being deformed by nonlinear effects, it corresponds to the minimum given by Eq. (18). In Fig. 1(b), one can also see an additional minimum at K close to 1.4. The origin of this minimum is not clear; it may be a purely nonlinear effect.

It may also be of interest to trace the evolution of the form of the driven oscillations with the increase of the amplitude ϵ from zero to the value ϵ_{cr} at fixed values of A and K. We shall display here characteristic results for the resonant values of K predicted by Eq. (11). For very small values of ϵ, e.g., for $\epsilon=0.001$ (Fig. 2), we observe linear driven oscillations. A solution of Eqs. (6) satisfying the particular initial conditions formulated above should take the form of a superposition of the driven and free oscillations. Due to a very weak nonlinearity (it is weak because of the smallness of the amplitude of the oscillations), the frequency of the free oscillations is slightly shifted from the value given by Eq. (10a). This produces a small frequency splitting between the driven and free components of the full law of motion, which gives rise to the long-period beatings clearly seen in Fig. 2. The results presented in Fig. 2 pertain to $A=\frac{2}{3}$; for $A=2$ the results are very similar.

With increase of ϵ, the oscillations become irregular (chaotic). As is illustrated by Fig. 3(a), they already seem quite irregular at $\epsilon=0.05$ (for $A=\frac{2}{3}$). For $A=2$, the oscillations are also chaotic at $\epsilon=0.05$ [Fig. 3(b)], although

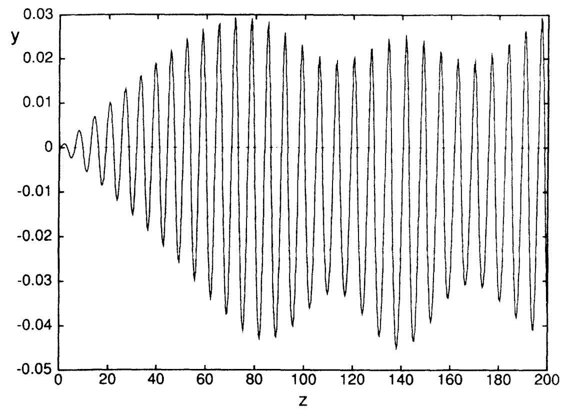

FIG. 2. Small-amplitude resonant driven oscillations of the separation y between the two components of the vector soliton in the case $A=\frac{2}{3}$, $\epsilon=0.001$.

they seem somewhat different from those shown in Fig. 3(a). As another example, in Fig. 4 we show chaotic oscillations, obtained for $A=2$ and the value of K given by Eq. (12) (i.e., at the point of the second resonance), at $\epsilon=0.15$. According to Eq. (16), this value is close to the corresponding splitting point.

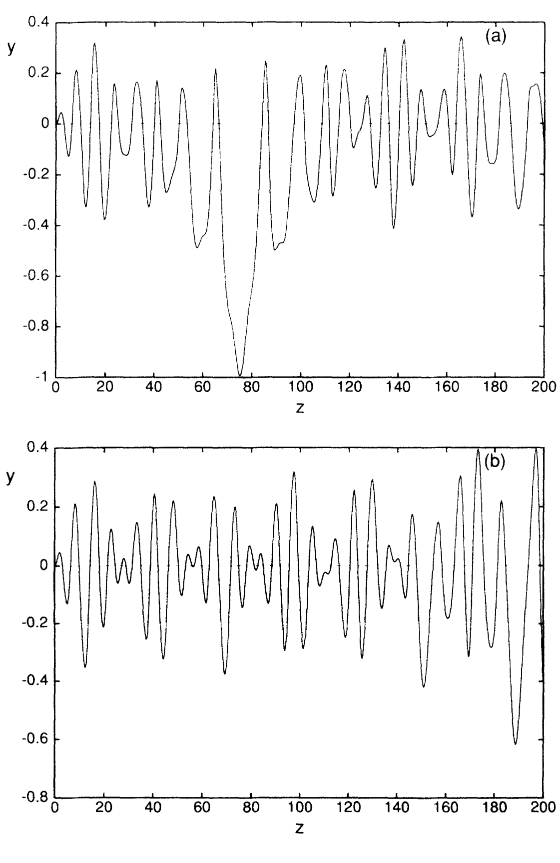

FIG. 3. Irregular resonant driven oscillations at $\epsilon=0.05$ for $A=\frac{2}{3}$ (a) and for $A=2$ (b).

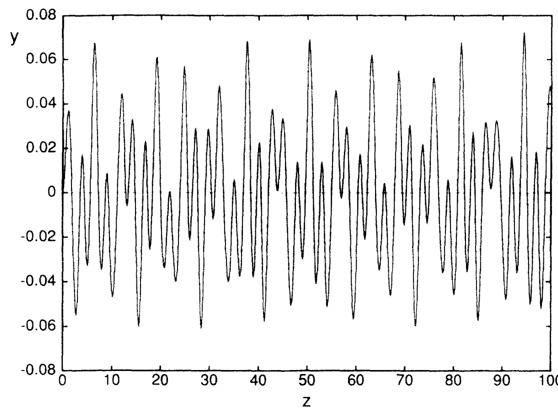

FIG. 4. Irregular driven oscillations in the case of resonance at the double frequency near the splitting point ($A=2$, $\epsilon=0.15$).

In Fig. 5 we show a typical example of the splitting of the vector soliton when the modulation amplitude ϵ slightly exceeds the critical value. We took the values $A=\frac{2}{3}$, $K=0.8$, and $\epsilon=0.13$, i.e., a point lying just above the resonant minimum of Fig. 1(b). As is seen in this figure, the vector soliton splits after performing several internal oscillations (cf. typical plots in Ref. [13]).

Lastly, as a final illustration of the importance of the *resonant* character of the birefringence modulation for the splitting of the vector soliton, in Fig. 6 we display the oscillations of the separation y between the centers of its two components produced by a *constant* force in Eq. (6a). To obtain Fig. 6, we solved Eqs. (6) with the term $\epsilon\cos z$ replaced by a constant ϵ. Obviously, from the viewpoint of the underlying partial differential equations (PDE's) (2), the constant force corresponds to the birefringence term $\delta(z)=\epsilon z$ growing linearly with z. In this run, we started from the same initial point as in the case of the modulated birefringence. We again took the values of the parameters close to those at which the plot in Fig. 1(b) has a minimum, i.e., $A=\frac{2}{3}$, $K=0.8$, and for the coefficient ϵ we took the value 0.058, which is close to the

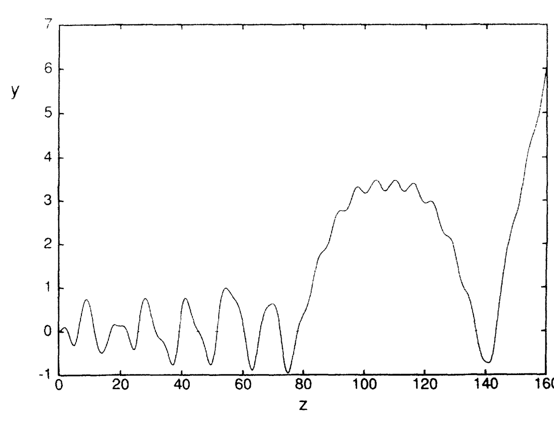

FIG. 5. Splitting of the vector soliton at $A=\frac{2}{3}$, $K=0.8$, and $\epsilon=0.13$.

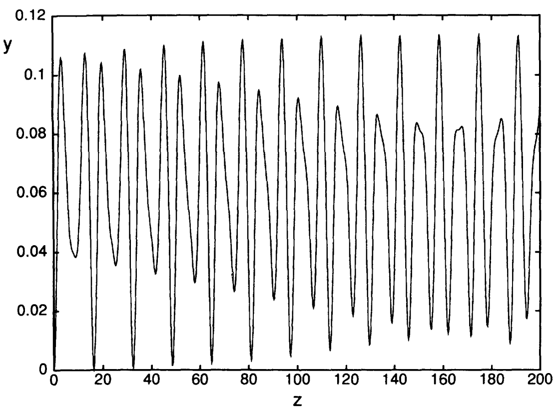

FIG. 6. Internal oscillations of the vector soliton produced by the linearly growing birefringence, corresponding to a constant force in Eq. (6a).

minimum amplitude (17) of the modulation which gives rise to splitting. As can be seen from Fig. 6, the constant force produces rather trivial internal vibrations of the vector soliton around a new equilibrium position shifted from the point $y=0$.

In addition to the physical values of the coupling constant A considered above, we have also done simulations for the value of A given by Eq. (13), when the two spatial eigenfrequencies (10) coincide with each other, but *not* with the driving frequency. For low values of K, off-resonance splitting is sometimes possible, but the corresponding critical values of ϵ are large. For instance, in the typical nonresonant case $K=\frac{1}{2}K_{\rm res}$, where $K_{\rm res}$ is the resonant value given by Eq. (14), the splitting was detected at $\epsilon_{\rm cr}\approx 0.40$. Then, for K between $0.6K_{\rm res}$ and $0.75K_{\rm res}$, the value of $\epsilon_{\rm cr}$ rapidly decreases. For instance, $\epsilon_{\rm cr}\approx 0.41$ at $K=0.6K_{\rm res}$ and $\epsilon_{\rm cr}\approx 0.23$ at $K=0.7K_{\rm res}$. At the point $K=K_{\rm res}$ [recall that, at the value of A given by Eq. (13) which we are considering now, this is the point of double resonance, in which the driving frequency coincides with the eigenfrequency (10a), and, simultaneously, twice the driving frequency coincides with the eigenfrequency (10b)], we have found $\epsilon_{\rm cr}\approx 0.09$, cf. Eqs. (15) and (17). However, at $K>K_{\rm res}$ the critical value of ϵ does not grow, as it did at the physical values $A=2$ and $A=\frac{2}{3}$ (see Fig. 1), but remains very small. This anomalous behavior is due to the small value of A (13) that we are dealing with now. From Ref. [6], at small A the value of the velocity needed for escape to occur is also small (see also Ref. [21]). Hence when A is small and K is large, the oscillations become large enough so that escape is possible. Let us emphasize once again that this anomalous behavior does not occur at the physical values of A, $\frac{2}{3}$, and 2, which prove to be sufficiently large to prevent escape in this trivial way.

V. CONCLUDING REMARKS

In this work, we have demonstrated, by means of an approximation based upon a variational technique, that a vector soliton in a birefringent nonlinear optical fiber with periodically modulated parameters is apt to split into a pair of simple solitons. It has been shown that this effect has a well-pronounced resonant character: the critical value of the modulation amplitude, at which the splitting sets in, has a deep minimum at the value of the soliton's energy, which gives rise to resonance between internal vibrations of the vector soliton and the periodic modulation. The location of the minimum of the critical modulation amplitude is fairly well predicted by the resonance condition, despite the fact that the eigenfrequency of the internal vibrations was obtained in the linear approximation for small oscillations, while splitting is a strongly nonlinear effect corresponding to very large oscillations. The analysis presented in this work was based upon the simplest model described by Eqs. (2). As was mentioned above, a more realistic model of a periodically modulated birefringent fiber may include more terms. However, we expect that the induced resonant splitting of the vector soliton in more sophisticated models will be similar to what has been obtained in the present model.

We have also briefly considered the case of constant driving force in Eqs. (6), which corresponds to the birefringence coefficient growing linearly with propagation distance. This situation may be novel from the physical viewpoint; however, simulations of Eqs. (6) have shown that the linearly growing birefringence is much less effective as a splitter of the vector soliton than the resonant periodic modulation.

It is noteworthy that a pair of nonlinear Schrödinger equations with the nonlinear coupling is an adequate model not only of a birefringent fiber, but also of the copropagation of two waves with different carrier wavelengths in a single-mode fiber [19]. In this case, the coupling constant in Eqs. (2) is $A=2$ (see also Ref [20]). It is well known that the copropagating waves may form two-component solitons, so that the results obtained in the present work may also be interpreted as the splitting of a two-wave soliton induced by the periodic modulation of the fiber.

Finally, it is relevant to mention that constant (unmodulated) birefringence can also split a vector soliton, but in this case the problem should be formulated in another way. Indeed, in our Eq. (6a) the effective driving force is proportional to the z derivative of the birefringence parameter $\delta(z)$ in Eqs. (2), i.e., it is equal to zero in the absence of modulation. On the other hand, constant birefringence, as was mentioned in Sec. I, may be transformed into an initial "relative velocity" of the two components of the vector soliton, which, of course, can split the soliton. Roughly speaking, this happens, provided that an effective "kinetic energy" generated by the initial relative velocity is larger than the "binding energy" of the vector soliton. This effect was investigated both numerically within the framework of the full equations (2) [3,10], and analytically by means of perturbation theory for two weakly coupled equations [21].

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